REFINEMENT OF THE RELATIONS OF THE KOL'SKII–HOPKINSON COMPOUND-ROD METHOD

V. M. Kornev

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In the following study, we will present a refined version of the relations used in the compound-rod method. The refined relations are constructed using the Rayleigh equation for the longitudinal vibration of a rod [1]. We also measure the acceleration on the surface of a measuring rod in one of the sections. This allows us to: 1) improve the compatibility of the parametrically assigned stress–strain function; 2) easily find the specimen strain rate; 3) correct the stresses in the specimen with allowance for the inertial component.

The well-established compound-rod method developed by Kol'skii and Hopkinson [2-4] makes it possible to obtain sufficiently reliable unidimensional dynamic stress–strain curves for appreciable strain rates. The classical relations of this method are based on a unidimensional theory of the propagation of elastic waves in rods and do not account for wave dispersion in actual rods or in three-dimensional objects:

$$\varepsilon=\frac{\partial u}{\partial x}=-\frac{1}{c}\frac{\partial u}{\partial t},\quad \sigma=E\varepsilon. \tag{1}$$

Here, $\varepsilon = \varepsilon(x, t)$ and $\sigma = \sigma(x, t)$ are the stress and strain; $u = u(x, t)$ is the displacement along the axis of the rod; x and t are the longitudinal coordinate and time; c is the velocity of propagation of unidimensional elastic waves in the rod, or the "rod" velocity – which in the classical theory is independent of wavelength: $c^2 = E/\rho$ (E and ρ are the elastic modulus and density of the material of the measuring rods).

We will propose a method of measurement for the rod which is almost the same as the classical method. The revised method is illustrated by Fig. 1, where 1 is the pressure-transmitting rod; 2 is the anvil rod; 3 is the specimen; 4 and 5 are strain gages; 6 is an acceleration transducer. The transducer is mounted on the surface of the anvil rod in the same section as the strain gage 5. We designate $\varepsilon_i(t)$, $\varepsilon_r(t)$ and $\varepsilon_t(t)$ as the strains in the incident, reflected, and transmitted waves, ℓ_s and ℓ as the length of the specimen and the distance from the ends of the rods to the strain gauges, and a as the radius of the rods and specimen. After completing the corresponding transformations [2] and allowing for ε_i, ε_r and ε_t, we use Eqs. (1) to obtain relations describing the behavior of the specimen:

$$\varepsilon_s=\varepsilon_s(t),\ \sigma_s=\sigma_s(t) \tag{2}$$

(t is a parameter). Equations (2) are a parametric representation of the curve σ_s–ε_s. The curve is not smooth because of the presence of dispersion [3]. Follensby and Franz [3] proposed that partial allowance be made for dispersion in the rod when analyzing measurements (see expressions (10) and (13) in [3], in which c = const); allowance was made for dispersion in [5] in the measurement analysis, but only for a layer. An analysis of the longitudinal vibrations of rods [1, 6] and a layer [5, 7] suggests that the process of the propagation of a strain pulse through a rod can be represented as a process taking place in a three-dimensional body in the following manner: a low-energy precursor propagated ahead with the velocity

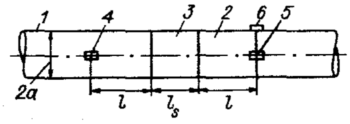

Fig. 1

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of the longitudinal waves; then the smooth main part of the pulse, distorted by dispersion, is transmitted (generally at the "rod" velocity); left behind by the main part of the pulse is a perturbation describing only shear strains (its velocity is the same as the velocity of the distortion waves). From a practical standpoint, most of the important data on the transmission of the pulse is obtained from its smooth main part. If the pulse has a nonremovable discontinuity (and let us assume that such conditions exist on the end), then the front of the nonremovable discontinuity moving with the "rod" velocity will be eroded [1, 6]. As was shown in [6], the propagation of the main part of the pulse is described by the Rayleigh equation to within the coefficient k. Assuming that the strain $\varepsilon_x = \partial u/\partial x$ is independent of the transverse coordinates y and z, we obtain

$$v = -\nu y \partial u/\partial x,\ w = -\nu z \partial u/\partial x,\ u = u(x,\ t) \tag{3}$$

(v and w are the displacements in the y and z directions and ν is the Poisson's ratio). After completing the corresponding transformations [1], we finally obtain

$$c^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} + \nu^2 k^2 \frac{\partial^4 u}{\partial x^2\, \partial t^2} = 0. \tag{4}$$

Here, k is the polar radius of inertia of the cross section, with $k^2 = a^2/2$ for a circular cross section (where a is the radius of the rod). The straight lines $ct \pm x$ are not characteristics of Eq. (4).

Let us examine the solutions in the form of transmitted waves

$$u_n = A_n \sin(q_n x \pm \omega_n t + \varphi_n) = A_n \sin q_n (x \pm c_n t + \varphi_n/q_n), \tag{5}$$

where A_n is the amplitude of a wave; q_n is the wave number ($q_n > 0$); ω_n is frequency; ϕ_n is the phase; $c_n = \omega_n/q_n$ is the velocity of the wave. We wrote the same solution in two different forms in Eq. (5), since they will both be used below. The wave number q_n characterizes the changeability of the solution along the rod. The wave moves with the velocity

$$c_n^2 = c^2/(1 + \nu^2 k^2 q_n^2). \tag{6}$$

It is obvious that $c_n \to c$ when $q_n \to 0$. Taking (5) and (6) into consideration, we obtain

$$\partial u/\partial x = \pm c_n^{-1} \partial u/\partial t, \tag{7}$$

where $c_n \neq c$ for $q_n > 0$. In contrast to classical equation (1), Eq. (7) accounts for wave dispersion.

Let us examine the displacements of the ends of rods u_1 and u_2 abutting the specimen. Here the subscripts 1 and 2 correspond to the pressure transmitter and anvil. Then $\partial u_2/\partial x = \varepsilon_t$, $\partial u_1/\partial x = \varepsilon_i + \varepsilon_r$. Considering that $\varepsilon_i - \varepsilon_r = \varepsilon_t$, we have

$$\partial(u_2 - u_1)/\partial x = -2\varepsilon_r. \tag{8}$$

Relations (8) and

$$\sigma_s = E\varepsilon_t \tag{9}$$

make it possible to account for the dispersion of the waves (see (6)).

We supplement the functions $\varepsilon_i(t)$, $\varepsilon_r(t)$, and $\varepsilon_t(t)$ obtained from measurements with the old periodic functions $\varepsilon_i^*(t)$, $\varepsilon_r^*(t)$, and $\varepsilon_t^*(t)$, then expanding the periodic functions $\varepsilon_r^*(t)$ and $\varepsilon_t^*(t)$ into Fourier series

$$\varepsilon_t^*(t) = \left.\frac{\partial u_t^*}{\partial x}\right|_{x=0} = \sum_{n=1}^{N} A_{nt} \sin \omega_n t = -\sum_{n=1}^{N} A_{nt} \sin(-q_n c_n t),$$
$$\varepsilon_r^*(t) = \left.\frac{\partial u_r^*}{\partial x}\right|_{x=0} = \sum_{n=1}^{N} A_{nr} \sin \omega_n t = \sum_{n=1}^{N} A_{nr} \sin q_n c_n t. \tag{10}$$

The coefficients A_{nt} and A_{nr} in (10) are calculated from standard formulas involving use of the first representation of the Fourier series. It should be noted that the point of reference x = 0 for each measurement of the layer and that this point coincides with the location of gauges 5 and 4. The number N is chosen on the basis of the following condition: the wavelength for N should be less than the rod radius (this condition must be satisfied to permit use of the Rayleigh equation (4) [1]). Let us proceed to the transmitted waves (see (5) and the second representation of the Fourier series in (10)). Considering that a transmitted wave propagates from left to right and a reflected wave from right to left, we obtain

$$\frac{\partial u_t^*}{\partial x}(x,t) = -\sum_{n=1}^{N} A_{nt}\sin q_n(x - c_n t), \tag{11}$$

$$u_r^*(x,t) = \sum_{n=1}^{N} \frac{A_{nr}}{q_n}\cos q_n(x + c_n t) + \frac{A_{0r}}{2},\ \frac{A_{0r}}{2} = -\sum_{n=1}^{N}\frac{A_{nr}}{q_n}.$$

In determining the constant A_{0r}, we used the condition that the rod was at rest before arrival of the strain pulse. We emphasize that there are different points of reference for the x coordinates in Eqs. (11), since they coincide with gauges 5 and 4. Let us consider allowing the dispersion in Eqs. (8-9) and (11). To determine the values on the ends of the specimen from the measurements of ε_i^*, $\partial u_t^*/\partial x$ and u_r^* (for which the gauges were located a distance ℓ from the ends), we introduce the coordinate transformation

$$t = t_1 \pm l/c. \tag{12}$$

A plus sign is chosen in (12) when the gauge is positioned before the end relative to the propagation of the disturbance in the incident wave, while a minus sign is used when the gauge is positioned past the end relative to the propagation of the disturbance in the reflected and transmitted waves. After completing the transformations, we obtain the following refined relations in the Kol'skii–Hopkinson compound-rod method:

$$\sigma_s = -E\sum_{n=1}^{N} A_{nt}\sin q_n\left[-l\left(1-\frac{c_n}{c}\right) - c_n t_1\right]; \tag{13}$$

$$\varepsilon_s = -\frac{2}{l_s}\left\{\sum_{n=1}^{N}\frac{A_{nr}}{q_n}\cos q_n\left[l\left(1-\frac{c_n}{c}\right) + c_n t_1\right] + \frac{A_{0r}}{2}\right\}. \tag{14}$$

The phase $\pm\ell(1 - c_n/c)$ in Eqs. (13-14) depends on the number n, since different waves have different velocities. Generally speaking, the value of $(1 - c_n/c)$ is small because $c \approx c_n$. However, the phase may also be large due to the length ℓ. Equations (13-14) for $\sigma_s = \sigma_s(t_1)$ and $\varepsilon_s = \varepsilon_s(t_1)$ account for dispersion in two ways: $c_i \neq c_j$ if $i \neq j$; the phase is independent of the number.

To improve the compatibility of the parametrically assigned stress–strain function, we will measure the acceleration of the surface of the measuring rod in the section where gauge 5 is located:

$$\frac{\partial^2 v}{\partial t^2} = -\nu a\frac{\partial^3 u}{\partial x\,\partial t^2} = \psi(t). \tag{15}$$

Recording the accelerations $\psi(t)$ makes it possible to refine the value of time corresponding to the first point of the function σ_s. Meanwhile, in the proposed variant, the accelerations can be measured in tests conducted either in compression or tension [2]. The rate of deformation in this case will obviously be calculated directly:

$$\dot{\varepsilon} = \frac{\partial}{\partial t}\frac{\partial u}{\partial x} = -\frac{1}{\nu a}\int_0^t \psi(t)\,dt.$$

Inertial forces become important in the case of high rates of deformation (of the order of 10^3-10^4 $\sec^{-1}$), and it is advisable to consider these forces when determining the actual curve σ_s–ε_s (see (3.19-3.20) and (3.23) in [2]). Let the measured acceleration on the surface of the rods be $\psi(t)$ (15). We supplement this function with the odd periodic function $\psi^*(t)$, thus obtaining the following (A_n are coefficients of the Fourier series)

$$\left.\frac{\partial^3 u^*}{\partial x\,\partial t^2}\right|_{x=0} = -\frac{\psi^*(t)}{\nu a} = -\frac{1}{\nu a}\sum_{n=1}^{N} A_n \sin\omega_n t = \frac{1}{\nu a}\sum_{n=1}^{N} A_n \sin(-q_n c_n t),$$

from which

$$\frac{\partial^2 u^*(x,t)}{\partial t^2} = \frac{1}{\nu a}\left\{\sum_{n=1}^{N}\frac{A_n}{q_n}\cos q_n(x-c_n t) + \frac{A_0}{2}\right\},\quad \frac{A_0}{2} = -\sum_{n=1}^{N}\frac{A_n}{q_n},$$

since the wave moves from left to right and the rod is at rest before the wave's arrival. Let us now take into account the dispersion of the waves in the problem (12). Let the geometric dimensions of the specimen satisfy the relation $\ell_s = \sqrt{3}\nu_s a$. Then the stresses in the specimen are calculated from the formula [2]

$$\sigma_s = -E\sum_{n=1}^{N} A_{nt}\sin q_n\left[-l\left(1-\frac{c_n}{c}\right)-c_n t_1\right] + \rho_s\frac{l_s}{2}\frac{\partial^2 u^*(t_1)}{\partial t^2}, \tag{16}$$

$$\frac{\partial^2 u^*(t_1)}{\partial t^2} = \frac{1}{\nu a}\left\{\sum_{n=1}^{N}\frac{A_n}{q_n}\cos q_n\left[-l\left(1-\frac{c_n}{c}\right)-c_n t_1\right] + \frac{A_0}{2}\right\}.$$

Thus, (14) and (16) are the final refined relations of the Kol'skii–Hopkinson method of compound rods.

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